$\frac{6-k\,\cancel{k<6}}{3} = 0$

$6-k\,\cancel{k<6} = 0$

$k<6$

Tentamen Integrerend project systeemtheorie, 23 januari 2012

The exam consists of 5 problems. Points for correct answers can be found below.

1. Consider the nonlinear system represented by the equations

$$\begin{aligned} \frac{d}{dt}x_1 &= x_2 \\ \frac{d}{dt}x_2 &= -x_1 + x_2^3 - u \end{aligned}$$

Here, u is an input.

a. Show that $(x_1^*(t), x_2^*(t), u^*(t)) = (\sin t, \cos t, (\cos t)^3)$ is a solution. klopt

→ **b.** Determine the linearization of the system around this solution.

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & \cos^2 t \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ -1 \end{pmatrix}u$$

2. Determine all values of $k \in \mathbb{R}$ for which the polynomial

$$p(s) := s^3 + 3s^2 + 3s + k$$

$0 < k < 9$

has all its roots in the open left half-plane.

3. Consider the system

$$\frac{d}{dt}x = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & 0 & 0 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ \beta \end{pmatrix} u.$$

Here, β is a given real number

a. Is this system internally stable? $\lambda=0$ $\lambda=-3{,}3$ $\lambda=0{,}3$ dus niet stabiel

$\lambda_1=0$ $\lambda_2<0$ $\lambda_3>0$

b. Determine all values of β for which the system is controllable. VP behalve $\beta=0$

→ **c.** For those values of β for which the system is not controllable, determine the uncontrollable eigenvalues.

$\lambda=0$

m.b.o. $\tilde{A} = (\;)$ $B(\;)$

4. Suppose we have two linear systems

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

and

$$\begin{aligned} \dot{x} &= \tilde{A}x + \tilde{B}u \\ y &= \tilde{C}x + \tilde{D}u \end{aligned} \tag{2}$$

with the same number of inputs and the same number of outputs. Assume the two systems are isomorphic, i.e., there exists an invertible matrix S such that $\tilde{A} = SAS^{-1}$, $\tilde{B} = SB$, $\tilde{C} = CS^{-1}$ en $\tilde{D} = D$.

a. Let R be the controllability matrix of system 1, and $\tilde{R}$ the one of system 2. Show that $\tilde{R} = SR$.

gewoon invullen

b. Let W be the observability matrix of system 1, and $\tilde{W}$ that of system 2. Show thatt $\tilde{W} = WS^{-1}$.

c. Prove the following: system 1 is controllable if and only if system 2 is controllable.

$n = \text{rank}(W) = \text{rank}(WS^{-1}) = \text{rank}(\tilde{W}) = n$

d. Prove the following: system 1 is observable if and only if system 2 is observable.

hetzelfde voor R en $\tilde{R}$

e. Prove the following: if system 1 is observable then there exists *exactly one* invertible matrix S such that $\tilde{A} = SAS^{-1}$, $\tilde{B} = SB$, $\tilde{C} = CS^{-1}$.

$\tilde{A}^2 = (SAS^{-1})(SAS^{-1})$

5. Consider the system

$$\frac{d}{dt}x = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix} x + \begin{pmatrix} 1 \\ 1 \end{pmatrix} u, \quad y = \begin{pmatrix} 1 & 1 \end{pmatrix} x$$

$(sI - A)^{-1} = \begin{pmatrix} \frac{1}{s-a} & \frac{1}{(s-a)^2} \\ 0 & \frac{1}{s-a} \end{pmatrix}$

a. Determine the transfer function of Σ.

b. Determine the impulse response function $K(t) = Ce^{At}B$

Points: (10 points for free)

$K(t) = 2e^{\alpha} + te^{\alpha}$

$H(s) = \frac{2s - 2a + 1}{(s-a)^2}$

Problem 1: 15
Problem 2: 15
Problem 3: 20
Problem 4: 25
Problem 5: 15

$\left(\begin{array}{cc|cc} s-a & -1 & 1 & 0 \\ 0 & s-a & 0 & 1 \end{array}\right) \to \left(\begin{array}{cc|cc} s-a & -1 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{s-a} \end{array}\right) \to \left(\begin{array}{cc|cc} s-a & 0 & 1 & \frac{1}{s-a} \\ 0 & 1 & 0 & \frac{1}{s-a} \end{array}\right)$

$\to \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{s-a} & \frac{1}{(s-a)^2} \\ 0 & 1 & 0 & \frac{1}{s-a} \end{array}\right)$